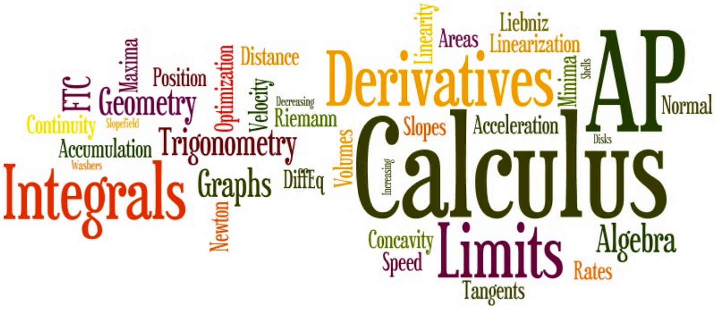

AP CALCULUS SUMMER PACKET ☺

DUE DATE: The beginning of class on the last class day of the first week of school.

The purpose of this assignment is to have you practice the mathematical skills necessary to be successful in AP Calculus. All of the skills covered in this packet are skills from Algebra 2 and Pre-Calculus. If you need to, you may use reference materials to assist you and refresh your memory (old notes, textbooks, Google, other online resources, etc.). While graphing calculators will be used in class, there are ***no calculators allowed on this packet***. You should be able to do everything without a calculator.

AP Calculus is a fast paced course that is taught at the college level. There is a lot of material in the curriculum that must be covered before the AP exam in May. Therefore, we cannot spend a lot of class time re-teaching prerequisite skills. This is why you have this packet. Spend some time with it and ***make sure you are clear on everything that is covered*** so that you will be successful in Calculus. Of course, you are always encouraged to seek help from your teacher if necessary.

This assignment will be collected and graded the last day of the first week of school. Be sure to ***fully complete the packet*** and ***show all appropriate work*** to support your answers**.** In addition, there will be a quiz on this material during the few weeks of school.

Good luck and have a great summer!

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Section I: Algebra Review

Identify the following statements as true or false.

1. $\frac{x+y}{2} = \frac{x}{2} + \frac{y}{2}$ _____

2. $\frac{1}{p+q} = \frac{1}{p} + \frac{1}{q}$ _____

3. $\frac{2k}{2x+h} = \frac{k}{x+h}$ _____

4. $3 \bullet \frac{a}{b} = \frac{3a}{b}$ _____

5. $3 \bullet \frac{a+b}{c} = \frac{3a+b}{c}$ _____

6. $\sqrt{a^2+b^2} = a+b$ _____

Identify the following statements as true or false over the set of real numbers. Give a counter example for any false statement.

7. $x^3 + 1 > x^3$ _____

8. $x^3 + x > x^3$ _____

9. $x^2 \geq 0$ _____

10. $x^2 \geq x$ _____

11. $2x \geq x$ _____

12. $\sqrt{x} \geq 0$ _____

13. $-x \leq 0$ _____

14. $\frac{1}{x} \leq x$ _____

15. $x \leq |x|$ _____

16. Solve $xy' + y + 1 = y'$ for y'.

17. Solve $\ln y = kt$ for y.

18. Factor: $y^3 + 27$

19. Factor: $x^2(x-1) - 4(x-1)$

16.______________

17.______________

18.______________

19.______________

Simplify each expression.

20. $\frac{(x^2)^3 x}{x^7}$ ________________

21. $\sqrt{x} \bullet \sqrt[3]{x} \bullet x^{\frac{1}{6}}$ ________________

22. $\frac{5(x+h)^3 - 5x^3}{h}$ ________________

23. $\frac{3(x+h)^2 - 3x^2}{h}$ ________________

24. $\dfrac{\dfrac{x^2-1}{x}}{\dfrac{x+1}{x^3}}$ ________________

25. $\dfrac{\dfrac{1}{x}+\dfrac{4}{x^2}}{3-\dfrac{1}{x}}$ ____________________

26. $\dfrac{\dfrac{a}{2x+h}-\dfrac{a}{2x}}{h}$ __________________

27. $\dfrac{1}{1-2a}-\dfrac{2}{1+2a}+\dfrac{6a+2}{4a^2-1}$ ______________________

Simplify, using factoring of binomial expressions. Leave answers in factored form.

Example:

$$\frac{(x+1)^3(4x-9)=(16x+9)(x+1)^2}{(x-6)(x+1)}=\frac{(x+1)^2\left[(x+1)(4x-9)-(16x+9)\right]}{(x-6)(x+1)}$$

$$=\frac{(x+1)^2(4x^2-5x-9-16x-9)}{(x-6)(x+1)}$$

$$=\frac{(x+1)^2(4x^2-21x-18)}{(x-6)(x+1)}$$

$$=\frac{(x+1)^2(4x+3)(x-6)}{(x-6)(x+1)}$$

$$=(x+1)(4x+3)$$

28. $(x-1)^3(2x-3)-(2x+12)(x-1)^2$ ____________________________

29. $\dfrac{(x-1)^2(3x-1)-2(x-1)\bullet 3}{(x-1)^4}$ ______________________________

30. $\dfrac{(x-1)^3(2x-3)-(4x-1)(x-1)^2}{(x-1)^2(2x-1)}$ ______________________________

Solve each equation or inequality for x over the set of real numbers.

33. $2x^4+3x^3-2x^2=0$ ___________________

34. $\dfrac{2x-7}{x+1}=\dfrac{2x}{x+4}$ _______________________

35. $\dfrac{3x+5}{(x-1)(x^4+7)}=0$ ____________________

36. $\sqrt{x^2-9}=x-1$ ____________________

37. $|2x-3|=14$ ____________________

38. $x^2-2x-8<0$ ____________________

Solve each of the systems.

39. $x+y=8$
$2x-y=7$ ____________________

40. $y=x^2-3x$
$y=2x-6$ ____________________

Section II: Pre-Calculus Review

Use your knowledge of the unit circle to evaluate each of the following. Leave your answers in radical form.

41. $\sin(30^\circ)$ _____

42. $\cos\dfrac{2\pi}{3}$ _____

43. $\tan 45^\circ$ _____

44. $\sin\left(-\dfrac{\pi}{6}\right)$ _____

45. $\tan\pi$ _____

46. $\csc\dfrac{5\pi}{6}$ _____

47. $\cos\left(90^\circ\right)$ _____

48. $\cos\dfrac{3\pi}{4}$ _____

49. $\tan\dfrac{\pi}{6}$ _____

50. $\cos^{-1}\left(\dfrac{1}{2}\right)$ _____

51. $\sin^{-1}\left(\dfrac{\sqrt{2}}{2}\right)$ _____

52. $\tan^{-1}(1)$ _____

Solve each trigonometric equation for $0\le x\le 2\pi$ *.*

53. $\sin x=\dfrac{\sqrt{3}}{2}$ ____________

54. $\tan^2 x=1$ ____________

55. $\cos\dfrac{x}{2}=\dfrac{\sqrt{2}}{2}$ ____________

56. $2\sin^2 x+\sin x-1=0$ ____________

For each trigonometric function identify the amplitude and any horizontal or vertical shifts from the basic function.

57. $y = \frac{1}{2}\cos\frac{x}{2} - 3$ amplitude:__________ period:_________ vertical shift:_________

58. $y = 2\sin(2x - \pi)$ amplitude:__________ period:_________ horizontal shift:________

59. $y = \tan 3x$ period:____________

Solve each exponential or logarithmic equation.

60. $5^x = 125$ __________

61. $8^{x+1} = 16^x$ __________

62. $81^{\frac{3}{4}} = x$ __________

63. $8^{\frac{-2}{3}} = x$ __________

64. $\log_2 32 = x$ __________

65. $\log_x \frac{1}{9} = -2$ __________

66. $\log_4 x = 3$ __________

67. $\log_3(x+7) = \log_3(2x-1)$ ____________

Expand each of the following using the laws of logs.

68. $\log_3 5x^2$ ________________________

69. $\ln\frac{5x}{y^2}$ _______________________

Complete each of the following using trigonometric identities and formulas.

70. $\sin\left(\frac{\pi}{2} - x\right) =$ ___________

71. $\sin^2 x + \cos^2 x =$ _________

72. $\sin 2u =$ __________________

73. $\tan x =$ _______________

74. $1 + \cot^2 x =$ ____________

75. $1 - \cos^2 x =$ _______________

76. A right triangle has a base of 5 and a hypotenuse of 7. Find the height.

Section III: Graphing Review

Sketch the following functions. State the domain and range of each. Draw and label your own axes.

77. $f(x) = x$

78. $f(x) = x^2$

79. $f(x) = x^3$

80. $f(x) = |x|$

81. $f(x) = [x]$ (Greatest integer function)

82. $f(x) = \frac{1}{x}$

83. $f(x) = \sqrt{x}$

84. $f(x) = e^x$

85. $f(x) = \ln x$

86. $f(x) = \sqrt{9 = x^2}$

87. $f(x) = \sin x$

88. $f(x) = \cos x$

89. $f(x) = \tan x$

90. $f(x) = \csc x$

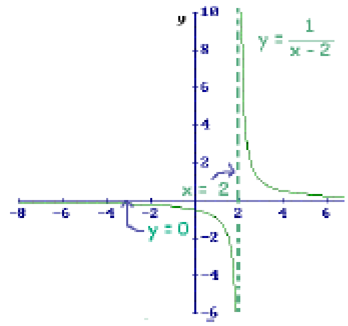
91. $f(x) = \sec x$

92. $f(x) = \cot x$

VERTICAL ASYMPTOTES

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote given the numerator does not equal 0 also (Remember this is called removable discontinuity).
Write a vertical asymptotes as a line in the form x =

Example: Find the vertical asymptote of $y = \frac{1}{x-2}$
Since when $x = 2$ the function is in the form 1/0 then the vertical line $x = 2$ is a vertical asymptote of the function.



44. $f(x) = \frac{1}{x^2}$

45. $f(x) = \frac{x^2}{x^2 - 4}$

46. $f(x) = \frac{2 + x}{x^2(1 - x)}$

47. $f(x) = \frac{4 - x}{x^2 - 16}$

48. $f(x) = \frac{x - 1}{x^2 + x - 2}$

49. $f(x) = \frac{5x + 20}{x^2 - 16}$

HORIZONTAL ASYMPTOTES

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is y = 0.

Example: $y=\frac{1}{x-1}$ (As x becomes very large or very negative the value of this function will approach 0). Thus there is a horizontal asymptote at $y=0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Exmaple: $y=\frac{2x^2+x-1}{3x^2+4}$ (As x becomes very large or very negative the value of this function will approach 2/3). Thus there is a horizontal asymptote at $y=\frac{2}{3}$.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Example: $y=\frac{2x^2+x-1}{3x-3}$ (As x becomes very large the value of the function will continue to increase and as x becomes very negative the value of the function will also become more negative).

Determine all Horizontal Asymptotes.

50. $f(x)=\frac{x^2-2x+1}{x^3+x-7}$

51. $f(x)=\frac{5x^3-2x^2+8}{4x-3x^3+5}$

52. $f(x)=\frac{4x^2}{3x^2-7}$

53. $f(x)=\frac{(2x-5)^2}{x^2-x}$

54. $f(x)=\frac{-3x+1}{\sqrt{x^2+x}}$ * Remember $\sqrt{x^2}=\pm x$

Solve for x:

55. $3^{3x+5} = 9^{2x+1}$

56. $\left(\frac{1}{9}\right)^x = 27^{2x+4}$

57. $\left(\frac{1}{6}\right)^x = 216$

LOGARITHMS

The statement $y = b^x$ can be written as $x = \log_b y$. They mean the same thing.
REMEMBER: A LOGARITHM IS AN EXPONENT

Recall $\ln x = \log_e x$

The value of e is 2.718281828… or $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

Example: Evaluate the following logarithms
$\log_2 8 = ?$
In exponential for this is $2^? = 8$
Therefore $? = 3$
Thus $\log_2 8 = 3$

Evaluate the following logarithms

58. $\log_7 7$

59. $\log_3 27$

60. $\log_2 \frac{1}{32}$

61. $\log_{25} 5$

62. $\log_9 1$

63. $\log_4 8$

64. $\ln\sqrt{e}$

65. $\ln\frac{1}{e}$

PROPERTIES OF LOGARITHMS

$\log_b xy = \log_b x + \log_b y$ $\quad \log_b \frac{x}{y} = \log_b x - \log_b y$ $\quad \log_b x^y = y \log_b x$ $\quad b^{\log_b x} = x$

Examples:

Expand $\log_4 16x$	Condense $\ln y - 2\ln R$	Expand $\log_2 7x^5$
$\log_4 16 + \log_4 x$	$\ln y - \ln R^2$	$\log_2 7 + \log_2 x^5$
$2 + \log_4 x$	$\ln \frac{y}{R^2}$	$\log_2 7 + 5\log_2 x$

Use the properties of logarithms to evaluate the following

66. $\log_2 2^5$

67. $\ln e^3$

68. $\log_2 8^3$

69. $\log_3 \sqrt[5]{9}$

70. $2^{\log_2 10}$

71. $e^{\ln 8}$

72. $9 \ln e^2$

73. $\log_9 9^3$

74. $\log_{10} 25 + \log_{10} 4$

75. $\log_2 40 - \log_2 5$

76. $\log_2 \left(\sqrt{2}\right)^5$

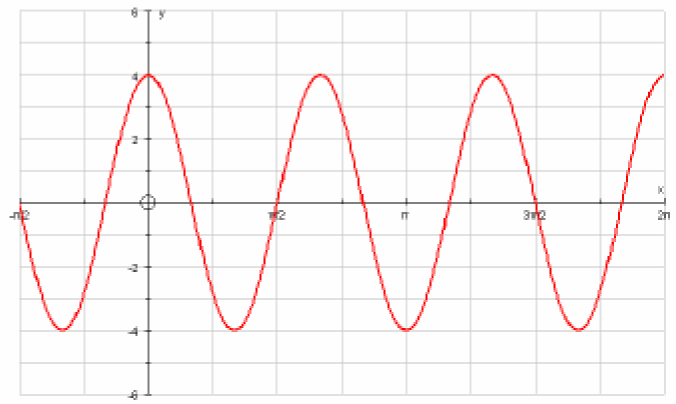
EVEN AND ODD FUNCTIONS

Recall:
Even functions *are functions that are symmetric over the y-axis.*
To determine algebraically we find out if $f(x) = f(-x)$
*(*Think about it what happens to the coordinate* $(x, f(x))$ *when reflected across the y-axis*)*

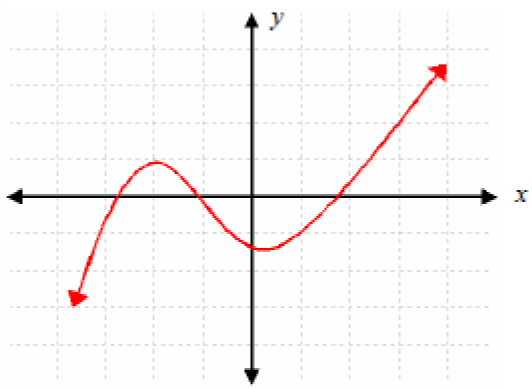
Odd functions *are functions that are symmetric about the origin.*
To determind algebraically we find out if $f(-x) = -f(x)$
*(*Think about it what happens to the coordinate* $(x, f(x))$ *when reflected over the origin*)*

State whether the following graphs are even, odd or neither, show ALL work.

77. ______________________



78. ______________________



79. ______________________
$f(x) = 2x^4 - 5x^2$

80. ______________________
$g(x) = x^5 - 3x^3 + x$

81. ______________________
$h(x) = 2x^2 - 5x + 3$

82. ______________________
$j(x) = 2\cos x$